PERGAMON

International Journal of Solids and Structures 37 (2000) 401–409

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Recent developments in anisotropic elasticity

T.C.T. Ting*

Department of Civil and Materials Engineering, College of Engineering, University of Illinois at Chicago, 842 West Taylor Street (M/C246), Chicago, IL 60607-7023, USA

Abstract

Anisotropic elasticity has been an active research subject for the last thirty years due to its applications to composite materials. There are essentially two formalisms for two-dimensional deformations of a *general* anisotropic elastic material. The Lekhnitskii formalism [Lekhnitskii, S.G., 1950. Theory of Elasticity of an Anisotropic Elastic Body. Gostekhizdat, Moscow (in Russian)] has been the favorite among the engineering community, while the newer Stroh formalism is well-known in the material sciences, applied mathematics and physics community. The Stroh formalism (Stroh, A.N., 1958. Dislocations and cracks in anisotropic elasticity. Phil. Mag. 3, 625–646.) is mathematically elegant and technically powerful. It began to be noticed by the engineering community in recent years, specially among the younger researchers. A comprehensive treatment of both formalisms and applications of the theory have been presented in a book by Ting. Since the appearance of the book in 1996, there have been several new developments in the theory and applications of anisotropic elasticity. We present here *new* results that have appeared since 1996. Only *linear anisotropic* elasticity is considered here; for nonlinear elasticity, the reader is referred to the book by Antman (Antman, S.S., 1995. Nonlinear Problems in Elasticity. Springer–Verlag, New York). © 1999 Published by Elsevier Science Ltd. All rights reserved.

The Stroh formalism (Stroh, 1958) does not give the stress σ_{ij} explicitly in a symmetric form. It does not give an explicit expression for the strain in a compact form. Mantic and Paris (1997) have recently derived the following explicit symmetric representation of stress,

$$\sigma = \mathrm{Re}\left\{\mathbf{B}\langle f_*\prime(z_*)q_*/B_2\rangle\mathbf{B}^T\right\}, \text{ (not for } \sigma_{33}), \tag{1}$$

in which Re stands for the real part, q_* is an arbitrary constant, $f_*(z_*)$ is an arbitrary function of $z_* = x_1 + p_* x_2$, $f'(z) = \mathrm{d}f(z)/\mathrm{d}z$, $\langle * \rangle$ $(* = 1, 2, 3)$ is a diagonal matrix and $\mathbf{B}$ is a 3×3 matrix that depends on elastic constants only (Ting, 1996a). Eq. (1) does not apply to σ_{33}. An alternate derivation that is

* Tel.: +1-312-996-2429; fax: +1-312-996-2426.
E-mail address: tting@uic.edu (T.C.T. Ting)

0020-7683/00/$ - see front matter © 1999 Published by Elsevier Science Ltd. All rights reserved.
PII: S0020-7683(99)00102-X

more elementary and transparent was presented by Ting (1998b), who also gave an explicit symmetric representation of strain as

$$\epsilon = \left[\mathrm{Re}\left\{\mathbf{A}\langle f_*{}'(z_*)q_*/B_{2*}\rangle\mathbf{B}^T\right\}\mathbf{K}\right]. \tag{2}$$

In the above, [●] stands for the symmetric part of the matrix, $\mathbf{A}$ is a 3×3 matrix that depends on elastic constants only, and

$$\mathbf{K} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{3}$$

By choosing $f'(z) = -2i$ and $q_*/B_{2*} = 1$, Eqs. (1) and (2) reduce to:

$$\sigma = \mathrm{Re}\{-2i\mathbf{B}\mathbf{B}^T\} = \mathbf{L}$$

and

$$\epsilon = -[\mathrm{Re}\{-2i\mathbf{A}\mathbf{B}^T\}\mathbf{K}] = -[\mathbf{SK}], \tag{4}$$

where $\mathbf{L}$ and $\mathbf{S}$ are two of the three Barnett–Lothe tensors (Barnett and Lothe, 1973). Thus, L_{ij} (except L_{33}) represent an admissible stress state. The strains ϵ_{ij} are related to S_{ij}. One can also show (Ting, 1998b) that the component $(\mathbf{LS})_{21}$ represents the strain energy density. The three Barnett–Lothe tensors $\mathbf{S}$, $\mathbf{H}$ and $\mathbf{L}$ appear frequently in the solutions to anisotropic elasticity.

The Stroh formalism reduces the problem to finding the eigenvalues p and the eigenvectors ξ of the eigenrelation (Ingebrigtsen and Tonning, 1969; Barnett and Lothe, 1973; Chadwick and Smith, 1977),

$$\mathbf{N}\xi = p\xi, \tag{5}$$

where $\mathbf{N}$ is a 6×6 real matrix that depends on elastic constants only. The six eigenvalues consist of three pairs of complex conjugates. If p_1, p_2 and p_3 are the eigenvalues with a positive imaginary part, the remaining three eigenvalues are the complex conjugates of p_1, p_2 and p_3. Let ξ_1, ξ_2 and ξ_3 be the eigenvectors associated with p_1, p_2 and p_3. The general solution for the displacement $\mathbf{u}$ and the stress function ϕ can be written in one equation as:

$$\mathbf{w} = \mathrm{Re}\{q_1 f_1(z_1)\xi_1 + q_2 f_2(z_2)\xi_2 + q_3 f_3(z_3)\xi_3\},\ \mathbf{w}^T = [\mathbf{u},\phi], \tag{6}$$

where q_1, q_2 and q_3 are arbitrary complex constants. There are three independent solutions. In the degenerate case in which $p_1 = p_2 \neq p_3$ and $\xi_1 = \xi_2$ there are only two independent solutions. Eq. (6) is not a general solution for the degenerate case. Isotropic elastic material is an example. Although the degenerate case, $p_1 = p_2 \neq p_3$, has been discussed (Ting and Hwu, 1988), a full investigation of all degenerate cases has only been done recently. Depending on the number of repeated eigenvalues p and the number of independent eigenvectors Eq. (5) has, the 6×6 matrix $\mathbf{N}$ can be classified into six groups as follows.

1. Simple $\mathbf{N}$ (the SP group). The matrix $\mathbf{N}$ is *simple* when it has three distinct eigenvalues p. The three associated eigenvectors are independent of each other. Most anisotropic elastic materials belong to this group.
2. Semisimple $\mathbf{N}$ (the SS group). The matrix $\mathbf{N}$ is *semisimple* when it has $p_1 = p_2 \neq p_3$ and there exist three independent eigenvectors. Examples can be found in Tanuma (1996) and Ting (1997a). Isotropic elastic materials belong to this group for surface waves.

3. Extraordinary semisimple **N** (the ES group). The matrix **N** is *extraordinary semisimple* when it has $p_1 = p_2 = p_3$ and there exist three independent eigenvectors. This group does not exist for surface waves (Ting, 1997a). However, it exists for steady state motion such as a moving line dislocation. It also exists for elastostatics if the strain energy is allowed to be *positive semidefinite*.
4. Degenerate **N** of the first kind (the D1 group). The matrix **N** is *degenerate of the first kind* when it has $p_1 = p_2 \neq p_3$ and there exist only two independent eigenvectors. Examples are given in Tanuma (1996).
5. Degenerate **N** of the second kind (the D2 group). The matrix **N** is *degenerate of the second kind* when it has $p_1 = p_2 = p_3$ and there exist only two independent eigenvectors. Isotropic elastic materials belong to this group in elastostatics. Other examples can be found in Ting (1994) and Tanuma (1996).
6. Extraordinary degenerate **N** (the ED group). The matrix **N** is *extraordinary degenerate* when it has $p_1 = p_2 = p_3$ and there exists only one independent eigenvector. That an ED group exists was proved in Ting (1996b) for elastostatics and for surface waves.

The general solution Eq. (6) is valid for the SP, SS and ES groups. For the D1 and D2 groups for which $p_1 = p_2$, it is replaced by (Ting and Hwu, 1988):

$$\mathbf{w} = q_1\xi_1 f_1(z_1) + q_2\frac{\mathrm{d}}{\mathrm{d}p_1}\{\xi_1 f_2(z_1)\} + q_3\xi_3 f_3(z_3). \tag{7}$$

For the ED group for which $p_1 = p_2 = p_3$, Eq. (6) is replaced by (Wang and Ting, 1997):

$$\mathbf{w} = q_1\xi_1 f(z_1) + q_2\frac{\mathrm{d}}{\mathrm{d}p_1}\{\xi_1 f_2(z_1)\} + q_3\frac{\mathrm{d}^2}{\mathrm{d}p_1^2}\{\xi_1 f_3(z_1)\}. \tag{8}$$

Thus, the ED group is the most degenerate in the mathematical sense. Isotropic elastic material, though the most degenerate in the physical sense, is not the most degenerate in the mathematical sense. Classifications of the materials based on the form of the solution for surface waves were presented in Ting and Barnett (1997). The classifications are important because the form of the solution depends on which group the material belongs to.

The physical meaning of Eqs. (7) and (8) is not clear for elastostatics. For surface waves in the half-space $x_2 > 0$, the displacement decays in proportion to $\mathrm{e}^{-\lambda x_2}$ ($\lambda > 0$) for Eq. (6). For Eqs. (7) and (8), it decays in proportion to $x_2\mathrm{e}^{-\lambda x_2}$ and $x_2^2\mathrm{e}^{-\lambda x_2}$, respectively (Ting, 1997b). Therefore, the decay of displacement is much slower for the ED group, followed by the D1 and D2 groups. How does a material change its **N** from one group to the other if we change the elastic constants continuously? This was studied in Ting (1998c).

The 6-vector ξ consists of two 3-vectors **a** and **b**. The original Stroh formalism (Eshelby et al., 1953) computes the 3-vector **a** from an eigenrelation. The 3-vector **b** is then determined from **a**. The Lekhnitskii formalism has no eigenrelation to speak of. As a result, it has been impossible to do the classifications with the Lekhnitskii formalism. A *hybrid* formalism has been proposed in Ting (1999a) that provides an eigenrelation for the 3-vector **b** (see also Yin, 1997; Barnett and Kirchner, 1997). The vector **a** is then determined from **b**. Thus, the hybrid formalism is a *dual* to the Stroh formalism. Application to the classifications of the 6×6 matrix **N** mentioned above shows that the hybrid formalism is much simpler and transparent. The classifications can be done without computing the eigenvalues p and the eigenvectors ξ, except for the SP and D1 groups for which the computation of p may be necessary. It is shown that the eigenvalues p can be computed explicitly for the SS, D2 and ED groups, and for certain materials in the D1 and SP groups. The explicit expression of the three Barnett–Lothe tensors **S**, **H** and **L**, especially **L** and **S**, can also be computed easily with the hybrid formalism (Ting, 1997c). It is clear that the hybrid formalism complements the Stroh formalism. For certain problems for which the Stroh formalism may be awkward and cumbersome to employ, the hybrid formalism offers an alternative. A good example in which a problem was solved by the Stroh formalism and by the hybrid formalism can be seen in Ting (1996c, 1999d). The simplification with the use of the hybrid formalism is striking.

The equivalence of the sextic equation in the Stroh formalism and the sextic equation in the Lekhnitskii formalism has been taken for granted. Barnett and Kirchner (1997) have shown directly that they are indeed equivalent. While the explicit expression of the sextic equation for the Lekhnitskii formalism can be easily derived, derivation of the explicit expression of the sextic equation of Stroh would be an unpleasant task as noted by Steeds (1973). Ting (1997c) presented an explicit expression of Stroh's sextic equation by using a *conversion formula* derived from Jacobi's theorem (see also Barnett and Chadwick, 1991). He converted the expression for $s_{\alpha\beta}'$ (the reduced elastic compliances in the Lekhnitskii's sextic equation) to an expression for $C_{\alpha\beta}$ (the elastic stiffnesses in the Stroh formalism). The conversion formula allows one to convert an expression for $s_{\alpha\beta}'$ to that for $C_{\alpha\beta}$ and vice versa. It will have wide applications.

There appear to be renewed interests in the singularities in anisotropic elastic wedges and composites, and in stress intensity factors. The fracture initiation in sharp notches was studied by Suwito et al. (1998). The free-edge singularity was investigated by Lin and Sung (1998) and Berger et al. (1998), in which the wedge angles in the two materials need not be $\pi/2$. On the problem of a crack terminating at an interface, Chen (1997) considered a crack that is normal to the interface of an orthotropic bimaterial. He showed that the stress intensity factor for mode I or II depends only on three composite elastic parameters. A crack that terminates at an interface obliquely was studied by Poonsawat et al. (1998), in which the interface can slide with friction. Chen and Hsu (1997) considered the difficult problem of an interfacial crack that is not a straight crack. It is a cusp, and is a generalization of the cusped crack in isotropic bimaterial considered by Wu (1994). Homentcovschi and Dascalu (1999) studied the more general case in which the crack is a lemon shape. The interfacial crack for which the displacement is prescribed on one surface while the traction is prescribed on the other surface of the crack was studied by Homulka and Keer (1995). On the subject of a curvilinear crack, Kattis (1999) studied a crack lying along an elliptic interface separating two dissimilar anisotropic elastic materials. The problem of the stress singularities at the apex of a wedge that consists of an arbitrary number of dissimilar anisotropic elastic wedges was studied by Ting (1997d). With the use of the *transfer matrix*, the size of the determinant that provides the stress singularities is independent of the number of the wedges. The classical paradox of Levy and Carothers on the critical wedge angle of an isotropic elastic wedge subject to a uniform traction or a concentrated couple was examined by Belov and Kirchner (1995) for a much more general wedge. The wedge can consist of any number of different anisotropic elastic wedges. Moreover, the elastic constants can depend on the polar angle.

A *cylindrically anisotropic* elastic material is a special inhomogeneous anisotropic elastic material. The stress–strain law for any material point is the same when it is referred to a cylindrical coordinate system. Examples of cylindrically anisotropic materials are tree trunks, carbon fibers (Christensen, 1994), certain steel bars, and manufactured composites (Rosenthal and Asimow, 1971). Lekhnitskii (1968) was the first to observe that the stress at the axis of a circular rod of cylindrically monoclinic material can be infinite when the rod is subject to a uniform radial pressure. A comprehensive re-examination of the Lekhnitskii problem for cylindrically orthotropic elastic material was given by Horgan and Baxter (1996). Ting (1996c) has considered a general cylindrically anisotropic elastic material, and has shown that the stress at the axis of the circular rod can also be infinite under a torsion or a uniform extension (see also Ting, 1999d). A general theory of cylindrically anisotropic radially inhomogeneous materials was presented by Alshits and Kirchner (1999a, 1999b). A remarkable phenomenon appears when one studies the stress singularities at the apex of a wedge of cylindrically anisotropic elastic material. Consider, for example, a cylindrically orthotropic elastic material under anti-plane deformations. The stress singularity at the wedge apex depends on one material parameter γ. For a given wedge angle 2α, no matter how small, one can choose a γ so that the stress at the wedge apex is infinite (Ting, 1998a). The wedge angle 2α can be any angle. It need not be larger than π, as is the case when the material is homogeneously isotropic or anisotropic. In the special case of a crack ($2\alpha = 2\pi$) there can be more than one stress singularity, some of them are stronger than the square root singularity. On the other hand, if $\gamma < 1/2$, there is no stress singularity at the wedge apex for any wedge angle, including the special case of a crack. The same remarkable nature prevails for the more complicated plane strain deformations (Ting, 1999b). The potential applications are obvious. If one can manufacture a structural element such that, at each wedge apex in the structure, the material is cylindrically orthotropic, the structure is *singularity-free* under any external loading. It is a *fail-safe* material.

Another remarkable result is related to the pressuring of a sphere. Assuming that the material is nonlinearly elastic but otherwise transversely isotropic with the radial direction being the axis of symmetry, Antman and Negron-Marrero (1987) have shown that the stress at the center of the sphere is infinite when the applied pressure exceeds a critical value. The same remarkable result was obtained by Ting (1999c) who assumed that the material is linear and anisotropic, but spherically uniform. It means that the stress–strain law referred to a spherically coordinate system is the same for each material point. The existence of the infinite stress at the center of the sphere is independent of the applied pressure. It depends only on one composite material parameter. What is more remarkable is that, even though the deformation is radially symmetric, the material at any point need not be transversely isotropic. The material can be *triclinic*, i.e., it need not possess a plane of symmetry.

The dependence of the solution on the number of elastic constants is of interest not only in theory, but also in applications. The well-known Dundurs constants for isotropic elastic bimaterials have been extended to monoclinic elastic bimaterials (Ting, 1995). The stress solution for plane strain deformations depends on two generalized Dundurs constants and the Stroh eigenvalues p_1 and p_2 in the two materials. An important result is that, for a two-phase composite for which the two materials are the same anisotropic elastic material but oriented differently from each other, the generalized Dundurs constants vanish. Other problems that involve the reduced dependence on elastic constants are discussed by Cherkaev et al. (1992), Thorpe and Jasiuk (1992), Dundurs and Markenscoff (1993), Yang and Ma (1998) and Chen (1997), mentioned earlier.

Although the contact problems for special anisotropic elastic materials have been considered before (see the chapter in this volume on Contact Mechanics by Barber and Ciavarella (1999)), the contact problems for a *general* anisotropic elastic material were studied only recently. Fan and Hwu (1996) studied the contact between a punch and a general anisotropic elastic half-plane. This is followed by a sliding punch on the half-plane with or without friction (Hwu and Fan, 1998a), contact of two dissimilar anisotropic elastic bodies (Hwu and Fan, 1998b), and a punch on a curvilinear hole boundary (Fan and Hwu, 1998). They also observed the mathematical analogy between the interface crack and the contact problems (Hwu and Fan, 1998c), two seemingly unrelated problems. On the subject of mathematical analogy Chen and Lai (1997) and Chen (1998) established a correspondence between a monoclinic piezoelectric material and a general anisotropic elastic material under two-dimensional deformations.

The presence of cracks, holes and inclusions in anisotropic elastic material is an important problem in applications. Most problems require a numerical solution such as using the Boundary Element Method (Hwu, 1998; Mantic and Paris, 1998). The essential ingredient in the BEM is the Green's function. The Green's functions for the infinite space, the half-space, bimaterials, elliptic inclusions, and cracks for anisotropic elastic materials and composites have all been obtained (see Ting, 1996a). New Green's functions for anisotropic elastic materials are obtained for a strip (Chiu and Wu, 1998), a wedge (Wu, 1998) and for piezoelectric, piezomagnetic and magnetoelectric media (Kirchner and Alshits, 1996). The success in obtaining these Green's functions is due to the powerful tool offered by the Stroh formalism.

Another example that owes its success to the Stroh formalism is the proof of Nix's theorem. Consider two line dislocations separated by a distance h but skewed with respect to each other by an angle. For an infinite anisotropic elastic medium, Orlov and Indenbom (1969) have shown that the net (integrated) interaction force of one dislocation exerted on the other is independent of the separation distance h. Nix (1997) computed numerically the net interaction force of one dislocation exerted on a skewed dislocation in an isotropic elastic half-space with a traction-free surface. His numerical results show consistently that the net interaction force is independent of h. Using the Stroh formalism, Barnett (1998) proved that the separation independence part of Nix's theorem holds for a general anisotropic elastic half-space with a traction-free surface. Further studies show that the separation independence remains valid for a bimaterial that consists of two dissimilar anisotropic elastic materials bonded together (Barnett et al., 1999) and for a half-space with a rigidly clamped surface or a slippery surface (Ting and Barnett, 1999). An interesting result for the infinite space is that the net interaction force vanishes if the Burgers vector of one of the two skew dislocations is in the direction of the shortest distance between the two skew dislocations. What is more amazing is that the net interaction force is independent of the presence of inclusions, voids or cracks (Ting, 1999e). Nix's theorem also holds for graded materials (Kirchner, 1999).

While the Lekhnitskii formalism is not as convenient as the Stroh formalism for most problems, a *modified* Lekhnitskii formalism or a *hybrid* formalism (Ting, 1999a) which is mathematically a *dual* to the Stroh formalism appears to have a great potential for the future research on anisotropic elasticity. This is so because the computation of the Stroh eigenvectors ξ in Eq. (5), which are needed in any solution, is much simpler by the hybrid formalism. The computation of the Barnett–Lothe tensors $\mathbf{L}$ and $\mathbf{S}$ is also simpler. An open question is whether there is a hybrid formalism for steady state motion such as surface waves in a general anisotropic elastic material. The solution for surface waves again relies on the Stroh eigenvectors. If the hybrid formalism can reproduce the Stroh eigenvectors for surface waves with simplicity, it would greatly benefit the analysis of surface waves.

There are other areas in anisotropic elasticity that remain to be explored. One is the problem of a hole in an anisotropic elastic material that is not an ellipse. Several papers on a hole of non-elliptic geometry have appeared in the literature but they are, mathematically, approximate solutions. The exact analysis of a non-elliptic hole has eluded researchers so far. It is a challenging problem. Another area is the three-dimensional problems. Some progress has been made in this direction. The Green's function for the three-dimensional space of general anisotropic elastic material was given in Ting and Lee (1997), in which the solution remains valid for the degenerate cases. The Green's function for the infinite space of transversely isotropic elastic material with the presence of body force was studied by Hanson (1998). Also studied was the solution for concentrated ring load in a full space or half-space of transversely isotropic material (Hanson and Wong, 1997). The Green's function for transversely isotropic piezoelectric material was discussed in Dunn and Wienecke (1996) for the infinite space and in Dunn and Wienecke (1999) for the half-space. The problem of a three-dimensional interfacial penny-shaped crack in anisotropic bimaterials was investigated by Qu and Xue (1998). Depending on the elastic constants, the pathological behaviour of interpenetration of the crack surfaces that exists in a two-dimensional crack may exist in a three-dimensional crack. Using the Radon transform, Wu (1999) has reduced the three-dimensional field equation to a two-dimensional one for which the Stroh formalism can be employed. This provides an extension of the Stroh formalism to three-dimensional problems of general anisotropic elastic materials. Like two-dimensional anisotropic elasticity, the problems of three-dimensional anisotropic elasticity should be simpler than the problems of three-dimensional isotropic elasticity because isotropic elastic material remains a *degenerate* material in three-dimensional deformations. The field of three-dimensional anisotropic elasticity is, indeed, wide open.

The references listed below are mostly for new results that appeared after 1996. References to important works that appeared before 1996 can be found in Ting's book (Ting, 1996a).

References

Alshits, V.I., Kirchner, H.O.K., 1999a. Elasticity of tree trunks. I. Fundamental equations for cylindrically anisotropic radially inhomogeneous elastic materials (submitted for publication).

Alshits, V.I., Kirchner, H.O.K., 1999b. Elasticity of tree trunks. II. Solutions for cylindrically anisotropic radially inhomogeneous elastic materials (submitted for publication).

Antman, S.S., Negron-Marrero, P.V., 1987. The remarkable nature of radially symmetric equilibrium states of aeolotropic nonlinearly elastic bodies. J. Elasticity 18, 131–164.

Barber, J.R., Ciavarella, M., 1999. Contact Mechanics. See the chapter in this volume.

Barnett, D.M., 1998. The net interaction force between two skew dislocations in an elastically anisotropic half-space. Scripta Materialia (the J.P. Hirth Symposium) 39, 371–378.

Barnett, D.M., Chadwick, P., 1991. The existence of one-component surface waves and exceptional subsequent transonic states of types 2, 4, and E1 in anisotropic elastic media. In: Wu, J.J., Ting, T.C.T., Barnett, D.M. (Eds.), Modern Theory of Anisotropic Elasticity and Applications. SIAM, Philadelphia, pp. 199–214 SIAM Proceedings Series.

Barnett, D.M., Kirchner, H.O.K., 1997. A proof of the equivalence of the Stroh and Lekhnitskii sextic equations for plane anisotropic elastostatics. Phil. Mag. A 76, 231–239.

Barnett, D.M., Lothe, J., 1973. Synthesis of the sextic and the integral formalism for dislocations, Greens function and surface waves in anisotropic elastic solids. Phys. Norv. 7, 13–19.

Barnett, D.M., Ting, T.C.T., Kirchner, H.O.K., 1999. The net interaction force between two skew dislocations in an anisotropic linear elastic bi-metallic medium. In: Taya, M., Arsenault, R.J. (Eds), Materials Science and Engineering A (in press).

Belov, A.Yu., Kirchner, H.O.K., 1995. Critical angles in bending of rotationally inhomogeneous elastic wedges. J. Appl. Mech. 62, 429–440.

Berger, J.R., Martin, P.A., Lien, J.P., 1998. Reduction of free-edge stress intensities in anisotropic bimaterials. Int. J. Fracture 91, 165–177.

Chadwick, P., Smith, G.D., 1977. Foundations of the theory of surface waves in anisotropic elastic materials. Adv. Appl. Mech. 17, 303–376.

Chen, C.-H., Hsu, J., 1997. The stress intensity factor of regularly perturbed interface cracks of anisotropic bimaterials. Int. J. Solids Structures 34, 1235–1253.

Chen, D.-H., 1997. Stress intensity factor for a crack normal to an interface between two orthotropic materials. Int. J. Fracture 88, 19–39.

Chen, T., 1998. Further correspondences between plane piezoelectricity and generalized plane strain in elasticity. Proc. Roy. Soc. London, A 454, 873–884.

Chen, T., Lai, D., 1997. An exact correspondence between plane piezoelectric and generalized plane strain in elasticity. Proc. Roy. Soc. London, A 453, 2689–2713.

Cherkaev, A.V., Lurie, K.A., Milton, G., 1992. Invariant properties of the stress in plane elasticity and equivalence classes of composites. Proc. Roy. Soc. London, A 438, 519–529.

Chiu, Y.T., Wu, K.C., 1998. Analysis for elastic strips under concentrated loads. J. Appl. Mech. 65, 626–634.

Christensen, R.M., 1994. Properties of carbon fibres. J. Mech. Phys. Solids 42, 681–695.

Dundurs, J., Markenscoff, X., 1993. Invariance of stresses under a change in elastic compliances. Proc. Roy. Soc. London, A 443, 289–300.

Dunn, M.L., Wienecke, H.A., 1996. Green's functions for transversely isotropic piezoelectric solids. Int. J. Solids Structures 33, 4571–4581.

Dunn, M.L., Wienecke, H.A., 1999. Half-space Green's functions for transversely isotropic piezoelectric solids. J. Appl. Mech. (in press).

Eshelby, J.D., Read, W.T., Shockley, W., 1953. Anisotropic elasticity with applications to dislocation theory. Acta Metall. 1, 251–259.

Fan, C.W., Hwu, C., 1996. Punch problems for an anisotropic elastic half-plane. J. Appl. Mech. 63, 69–76.

Fan, C.W., Hwu, C., 1998. Rigid stump indentation on a curvilinear hole boundary of an anisotropic elastic body. J. Appl. Mech. 65, 389–397.

Hanson, M.T., 1998. Some observation on the potential functions for transversely isotropy in the presence of body forces. Int. J. Solids Structures 35, 3793–3813.

Hanson, M.T., Wong, Y., 1997. Concentrated ring loadings in a full space or half-space: Solutions for transverse isotropy and isotropy. Int. J. Solids Structures 34, 1379–1418.
Homentcovschi, D., Dascalu, C., 1999. Uniform asymptotic solutions for lamellar inhomogenieties in anisotropic elastic solids. SIAM J. Applied Mathematics (in press).
Homulka, T.A., Keer, L.M., 1995. A mathematical solution of a special mixed-boundary-value problem of anisotropic elasticity. Q. J. Mech. Appl Math. 48, 635–658.
Horgan, C.O., Baxter, S.C., 1996. Effects of curvilinear anisotropy on radially symmetric stresses in anisotropic linear elastic solids. J. Elasticity 42, 31–48.
Hwu, C., 1998. A new BEM for two-dimensional anisotropic elastic solids containing multiple holes, cracks and inclusions. In: Bush, M.B. (Ed.), Discontinuous Materials and Structures. WIT Press, Computational Mechanics Publications, Southampton, UK, pp. 53–95.
Hwu, C., Fan, C.W., 1998a. Sliding punches with or without friction along the surface of an anisotropic elastic half-plane. Q. J. Mech. Appl. Math. 51, 159–177.
Hwu, C., Fan, C.W., 1998b. Contact problems of two dissimilar anisotropic elastic bodies. J. Appl. Mech. 65, 580–587.
Hwu, C., Fan, C.W., 1998c. Solving the punch problems by analogy with the interface crack problems. Int. J. Solids Structures 30, 3945–3960.
Ingebrigtsen, K.A., Tonning, A., 1969. Elastic surface waves in crystal. Phys. Rev. 184, 942–951.
Kattis, M.A., 1999. Non-planar interfacial cracks in anisotropic bimaterials. Int. J. Fracture (in press).
Kirchner, H.O.K., 1999. Skew line defects in graded materials. Phil. Mag. Letter 79, 209–216.
Kirchner, H.O.K., Alshits, V.I., 1996. Elastically anisotropic angularly inhomogeneous media II, The Green's function for piezoelectric, piezomagnetic and magnetoelectric media. Phil Mag. 74, 861–885.
Lekhnitskii, S.G., 1968. Anisotropic Plates, 2nd ed. Gordon and Breach, New York (S.W. Tsai, T. Cheron, Transl.).
Lin, Y.Y., Sung, J.C., 1998. Stress singularities at the apex of a dissimilar anisotropic wedge. J. Appl. Mech. 65, 454–463.
Mantic, V., Paris, F., 1997. Symmetric representation of stresses in the Stroh formalism and its applications to a dislocation and a dislocation dipole in an anisotropic elastic medium. J. Elasticity 47, 101–120.
Mantic, V., Paris, F., 1998. Integral kernels in the 2-D Somigliana displacement and stress identities for anisotropic materials. Computational Mechanics 22, 77–88.
Nix, W.D., 1997. Private communications with D.M. Barnett.
Orlov, S.S., Indenbom, V.L., 1969. The interaction of non-parallel dislocations in an anisotropic medium. Kristallografiya 14, 780–783.
Poonsawat, P., Wijeyewickrema, A.C., Karasudhi, P., 1998. Stress singularity analysis of a crack terminating at an interface of an anisotropic layered composite. J. Appl. Mech. 65, 829–836.
Qu, J., Xue, Y., 1998. Three-dimensional interface cracks in anisotropic bimaterials: The non-oscillatory case. J. Appl. Mech. 65, 1048–1055.
Rosenthal, D., Asimow, R.W., 1971. Introduction to Properties of Materials. Van Nostrand Reinhold, New York.
Steeds, J.W., 1973. Anisotropic Elasticity theory of Dislocations. Clarendon Press, Oxford.
Stroh, A.N., 1958. Dislocations and cracks in anisotropic elasticity. Phil. Mag. 3, 625–646.
Suwito, W.M., Dunn, M.L., Cunningham, S.J., 1998. Fracture initiation at sharp notches in single crystal silicon. J. Appl. Phys. 83, 3574–3582.
Tanuma, K., 1996. Surface impedance tensors of transversely isotropic elastic materials. Q. J. Mech. Appl. Math. 49, 29–48.
Thorpe, M.F., Jasiuk, I., 1992. New results in the theory of elasticity for two-dimensional composites. Proc. Roy. Soc. London, A 438, 531–534.
Ting, T.C.T., 1994. On anisotropic elastic materials that possess three identical Stroh eigenvalues as do isotropic materials. Q. Appl. Math. 52, 363–375.
Ting, T.C.T., 1995. Generalized Dundurs constants for anisotropic bimaterials. Int. J. Solids Structures 32, 483–500.
Ting, T.C.T., 1996a. Anisotropic Elasticity: Theory and Applications. Oxford University Press.
Ting, T.C.T., 1996b. Existence of an extraordinary degenerate matrix **N** for anisotropic elastic materials. Q. J. Mech. Appl. Math. 49, 405–417.
Ting, T.C.T., 1996c. Pressuring, shearing, torsion and extension of a circular tube or bar of cylindrically anisotropic material. Proc. Roy. Soc. London, A 452, 2397–2421.
Ting, T.C.T., 1997a. On extraordinary semisimple matrix $\mathbf{N}(v)$ for anisotropic elastic materials. Q. Appl. Math. 55, 723–738.
Ting, T.C.T., 1997b. Surface waves in anisotropic elastic materials for which the matrix $\mathbf{N}(v)$ is extraordinary degenerate, degenerate, or semisimple. Proc. Roy. Soc. London, A 453, 449–472.
Ting, T.C.T., 1997c. New explicit expression of Barnett–Lothe tensors for anisotropic linear elastic materials. J. Elasticity 47, 23–50.
Ting, T.C.T., 1997d. Stress singularities at the tip of interfaces in polycrystals. In: Rossmanith, H.P. (Ed.), Proc. First Int. Conf. on Damage and Failure of Interfaces. Balkema Pub, Rotterdam, Netherlands, pp. 75–82.

Ting, T.C.T., 1998a. The remarkable nature of cylindrically anisotropic elastic materials exemplified by an anti-plane deformations. J. Elasticity 49, 269–284.

Ting, T.C.T., 1998b. Symmetric representation of stress and strain in the Stroh formalism and physical meaning of the tensors **L**, **S**, $\mathbf{L}(\theta)$ and $\mathbf{S}(\theta)$. J. Elasticity 50, 91–96.

Ting, T.C.T., 1998c. Change between the six groups of anisotropic elasticity solutions by a continuous change of elastic constants. In: DeSanto, J.A. (Ed.), Proc. 4th Int. Conf. on Mathematical and Numerical Aspects of Wave Propagation. SIAM, Philadelphia, pp. 157–161 SIAM Proceedings Series.

Ting, T.C.T., 1999a. A modified Lekhnitskii formalism a la Stroh for anisotropic elasticity and classifications of the 6×6 matrix **N**. Proc. Roy. Soc. London, A 455, 69–89.

Ting, T.C.T., 1999b. The remarkable nature of cylindrically orthotropic elastic materials under plane strain deformations. Q. J. Mech. Appl. Math. 52, 387–404.

Ting, T.C.T., 1999c. The remarkable nature of radially symmetric deformation of spherically uniform linear anisotropic elastic solids. J. Elasticity 53, 47–64.

Ting, T.C.T., 1999d. New solutions to pressuring, shearing, torsion, and extension of a cylindrically anisotropic elastic circular tube or bar. Proc. Roy. Soc. London A455, 3533–3548.

Ting, T.C.T., 1999e. Independence on the presence of cracks or inclusions of the net interaction force acting on a dislocation by a skewed line singularity. Mathematics and Mechanics of Solids (submitted for publication).

Ting, T.C.T., Barnett, D.M., 1997. Classifications of surface waves in anisotropic elastic materials. Wave Motion 26, 207–218.

Ting, T.C.T., Barnett, D.M., 1999. On Nix's theorem for two skew dislocations in anisotropic elastic half-spaces and bimaterials. Mathematics and Mechanics of Solids (in press).

Ting, T.C.T., Hwu, C., 1988. Sextic formalism in anisotropic elasticity for almost non-semisimple matrix **N**. Int. J. Solids Structures 24, 65–76.

Ting, T.C.T., Lee, V.-G., 1997. The three-dimensional elastostatic Green's function for general anisotropic linear elastic solids. Q. J. Mech. Appl. Math. 50, 407–426.

Wang, Y.M., Ting, T.C.T., 1997. The Stroh formalism for anisotropic materials that possess an almost extraordinary degenerate matrix **N**. Int. J. Solids Structures 34, 401–413.

Wu, C.H., 1994. Regularly and singularly perturbed crack. Q. Appl. Math. 52, 529–543.

Wu, K.C., 1998. The elastic fields of a line force or dislocation in an anisotropic wedge. Int. J. Solids Structures 35, 3483–3495.

Wu, K.C., 1999. Generalization of the Stroh formalism to three-dimensional anisotropic elasticity. J. Elasticity 51, 213–225.

Yang, W., Ma, C.-C., 1998. Orthotropic transform for planar anisotropic elasticity and reduced dependence of elastic constants. Proc. Roy. Soc. London, A 454, 1843–1855.

Yin, W.L., 1997. A general analysis method for singularities in composite structures. In: Proc. 38th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, pp. 2239–2246.